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On essential pseudo principally-injective modules

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ABSTRACT

Pseudo-injectivity is a generalization of quasi- injectivity. An essential Pseudo-injective module wasintroduced by R. Jaiswal, P.C. Bharadwaj and S. Wongwai [7]. This paper is generalization of above notion with new properties.

Keywords: *Pseudo-injective module, essential pseudo-injective module, essential –Principally submodule and essential pseudo-Principally-injective module. .*

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I. INTRODUCTION

Through this paper, by a ring R we always mean as associative with identity and every R-module is unitary. The notion principally injective module was introduced by Camollo [9]. R.Jaiswal and P.C. Bharadwaj studied the structure of essentially pseudo principally injective modules. A submodule K of M is called essential submodule if $K \bigcap L \neq 0$ for every nonzero submodule L of M. In other words K ∩ N = 0 ⇒ K = 0 (briefly; $K \leq^e M$). In this case M is called essential extension of K. A monomorphism $f : K \to M$ is said to be essential if $\operatorname{imf} \leq^e M$. ($S = \operatorname{End}_R(M)$ Denotes endomorphism ring of M). An R-module M is said to be principally injective if for each R-homomorphism $\alpha : aR \to M$ such that a ∈R, extends to R. An R-module M is said to be pseudo injective if for every R-monomorphism $\beta : A \to M$ and $\alpha : A \to M$, there exists $\gamma \in \operatorname{End}(M)$ such that $\beta = \gamma.\alpha$ An R-module M is said to be pseudo M- injective if for every submodule A of M, any monomorphism $\alpha : A \to M$ can be extends to a homomorphism $\beta \in \operatorname{Hom}(M, N)$.An R-module M is said to be essential pseudo injective if for every sub module A, any essential monomorphism $\alpha : A \to M$ and monomorphism $\beta : A \to M$ there exists $h \in \operatorname{End}(M)$ such that $\alpha = h.\beta$.An R-module N is said to be pseudo M-P-injective if for any s∈S = End(M) and every monomorphism from s(M) to N, can be extended to a homomorphism from M to N. An R-module N is said to be essential pseudo M-P-injective if for any principally essential submodule s(M) of M, any monomorphism $f : s(M) \to N$ can be extended to some $g \in \operatorname{Hom}(M, N)$

II. MAIN RESULTS

Proposition.2.1. Let N be a module. Then following statements are equivalent:

1. If N is essential pseudo injective.
2. For every essential monomorphism $\beta : s(M) \to M$ and $\alpha : s(M) \to N$, where N embeds in M, there exists $\gamma \in \operatorname{Hom}_R(N, M)$ such that $\beta = \gamma.\alpha$.
3. For every essential monomorphism $\beta : s(M) \to M$ and $\alpha : s(M) \to N$, where N is a submodule of M, there exists $\gamma \in \operatorname{Hom}_R(N, M)$ such that $\beta = \gamma.\alpha$.
4. Every essential monomorphism $\varphi : N \to M$ where N is a submodule of M, can be extended to End(M).

Proof. (1) ⇒ (2) Let $\beta : s(M) \to M$ and $\alpha : s(M) \to N$, are essential monomorphisms. There exists $\gamma_1 : N \to M$. It is easy to check that $\gamma_1.\alpha : s(M) \to M$ is monic. Then there exists $\gamma_2 \in \operatorname{End}(M)$ Such that $\gamma_2 \gamma_1.\alpha = \beta$, Since M is essential pseudo injective. Let $\gamma_2 \gamma_1 = \gamma : N \to M$.Then $\beta = \gamma.\alpha$.

(2)⇒(3)⇒(4) clearly.

(4)⇒(1) Let $\beta : s(M) \to M$ and $\alpha : s(M) \to N$, be essential monomorphisms. Then $\alpha : s(M) \to Im(\alpha)$ is an isomorphism, so there exists $\alpha^{-1} : Im(\alpha) \to s(M)$ such that $\alpha^{-1}.\alpha = 1_{s(M)}$.Then $\beta.\alpha^{-1}$: $Im(\alpha) \to M$ is monic. Therefore there exists $\gamma \in End_R(M)$ such that $\gamma|_{Im\alpha} = \beta.\alpha^{-1}$, for every $a \in s(M)$, $\gamma.\alpha(a) = \beta.\alpha^{-1}.\alpha(a) = \beta(a)$, i.e. $\gamma.\alpha = \beta$.

Proposition.2.2. Let M_R be an essential pseudo injective module. Then
1) Every essential monomorphism $\alpha \in End_R(M)$ splits.
2) For every essential monomorphism $\beta : s(M) \to M$ and $\alpha : s(M) \to s(M)$, There exists $\gamma \in Hom_R(s(M), M)$ such that $\beta = \gamma.\alpha$.
3) Every essential monomorphism $\alpha \in Hom_R(M,N)$, where N embeds in M splits.

Proof.
1) Obvious
2) Let $\beta : s(M) \to M$ and $\alpha : s(M) \to s(M)$ be monomorphisms. Then s(M) embeds in M.
So there exists $\gamma \in Hom_R(s(M), M)$ such that $\beta = \gamma.\alpha$. by (1.1).
3) Let $\alpha \in Hom_R(M,N)$ be an essential monomorphism. Then for $\alpha : M \to N$ and $1_M : M \to M$, There exists $\beta \in Hom_R(N,M)$ such that $1_M = \beta.\alpha$ by (1.1).

Proposition.2.3.Let $(U_a)_{a\in I}$ be an indexed set of right R-modules. If $\oplus_I U_a$ essential pseudo injective, then the every essential monomorphism $\beta : s(M) \to U_a$ and $\alpha: s(M) \to U_b$ where $a, b \in I$, there exists $\gamma \in Hom_R(U_a, U_b)$ such that $\beta = \gamma.\alpha$.
Proof. Let $(U_a)_{a\in I}$ be an indexed set of right R-modules. Let $\beta : s(M) \to U_a$ and $\alpha: s(M) \to U_b$ be essential monomorphisms . For $i_a\beta : s(M) \to \oplus_I U_a$ and $\alpha: s(M) \to U_b$, where i_a is essential monomorphism from U_a to $\oplus_I U_a$ and the images $i_a s(M)$ are in $\oplus_I U_a$, there exists γ $\bar{\gamma} \in Hom_R(U_b, \oplus_I U_a)$ such that $i_a\beta = \bar{\gamma}.\alpha$ by (1.1). Let $\gamma = \pi_a. \bar{\gamma} : U_a \to U_a$. then $\gamma.\alpha = \pi_a. \bar{\gamma}\alpha = \pi_a. i_a\beta = \beta$.
Corollary.2.1. Every direct summand of essential pseudo module is also essential pseudo injective module.

III. ESSENTIAL PRINCIPALLY PSEUDO-INJECTIVE MODULE
(EPP-injective module)
An R-module M is called essential principally pseudo- injective if each essential monomorphism from an essential principal submodule of M to M can be extended to an endomorphism of M to M.
Let M be an R-module. We Write $l_M(m) = \{m \in M : mr = 0, \forall r \in R\}$ and $r_M(m) = \{r \in R : mr = 0, \forall m \in M\}$ for each $X \subset M$, the fined by right (left) annihilator of x in R is defined by $r_R(X) = \{r \in R : xr = 0, \forall x \in X\}$

$l_R(X) = \{r \in R : xr = 0, \forall x \in X\}$

$A_m = \{n \in M : r_R(n) = r_R(m), \forall m \in M\}$,
$S_{(\alpha,m)} = \{\beta \in S : ker\beta \cap mR = ker\alpha \cap mR, \forall m \in M\}$
$B_m = \{\alpha \in S : ker\alpha \cap mR = 0, \forall m \in M\}$.

Proposition3.1. For a given module M with
S = $End_R(M)$, the following conditions are equivalent for an element $m \in M$:
1. M is EPP-injective module.
2. $A_m = B_m m$
3. If $A_m = A_n$, then $B_m m = B_n n$.
4. For every essential monomorphismα: $mR \to M$ and β: $mR \to M$, there exists $\gamma \in End_R(M)$ such that $\alpha = \gamma.\beta$.

Proof. (1) ⇒ (2) let M be EPP-injective module. If n is an element, then $A_m = A_n$.Consider the mapping α: $mR \to M$ defined by $\alpha(mr) = nr$. Let $mr_1 = mr_2$ for all $r_1, r_2 \forall R$, so $mr_1 - mr_2 = 0$
$\Rightarrow \alpha(m(r_1 - r_2)) = 0 \Rightarrow n(r_1 - r_2) = 0 \Rightarrow nr_1 = n r_2$. Since M is EPP-injective, so α is monomorphism, can be extended M to M. Then $s(m) = \alpha(m) = n = sm$, where $s \in B_m$. Conversely; If $sm \in B_m m$, then $s \in B_m$ i.e. $\{kers \cap mR\} = 0$. It is clear that $r_R(sm) \supseteq r_R(m)$. If $r \in r_R(sm)$, then $smr = 0$, so $mr \in kers \cap mR = 0$, and $r \in r_R(m) \Rightarrow mr = 0$. Therefore $rR(sm) = r_R(m)$. Then $sm \in A_m$.
(2) ⇒ (3) Let $A_m = A_n$. Then $A_m = B_m m$ and $A_n = B_n n$. So $B_m m = B_n n$.

(3) $\Rightarrow$ (4) Let α: mR $\to$ M and β: mR $\to$ M be essential monomorphisms. Then $r_R(\beta m) = r_R(\alpha m)$. So $A_{\alpha m} = A_{\beta m}$, and $B_{\alpha m}\ \alpha m= B_{\beta m}\beta m$ by (3). Because $\{\ker s 1_M \cap \alpha mR\} = 0 \Rightarrow 1_M \in B_{\alpha m}$. Then $\alpha m \in B_{\beta m}\beta m$. There exists $\gamma \in B_{\beta m}$ such that $\alpha = \gamma.\beta$.

(4) $\Rightarrow$ (1) Put $\beta = i_{mR}$ in (4).

Proposition.3.2. Let M be EPP-injective module with S = $End_R(M)$. Then $S_{(\alpha,m)} = B_{\alpha m}\ \alpha + l_S(M)$.

Proof. If $\beta \in S_{(\alpha,m)}$, then $\ker\beta \cap mR = \ker\alpha \cap mR$, for all $m \in M$, Since $r_R(\alpha m) = r_R(\beta m)$.
If $\alpha(m)r = 0$
$\Rightarrow mr \in \ker\alpha \cap mR, = \ker\beta \cap mR$, so $\beta(m)r = 0$. If $\beta(m)r_1 = 0 \Rightarrow mr_1 \in \ker\beta \cap mR = \ker\alpha \cap mR$, so $\alpha(m)r_1 = 0$. Thus $\beta m \in B_{\alpha m}\ \alpha m$ by 2.1. This shows $\beta m = b\alpha m$ for all $b \in B_{\alpha m}$. this means $\beta\text{-}b\alpha \in l_S(m)$. $\Rightarrow \beta \in b\alpha + l_S(m)$. Conversely; Let $b\alpha +s \in B_{\alpha m}\ \alpha + l_S(m)$, with $b \in B_{\alpha m}$, $s \in l_S(m)$. If $mr \in \ker(b\alpha +s) \cap mR \Rightarrow (b\alpha +s)(mr) = b\alpha mr + smr = bb\alpha mr = 0 \Rightarrow \alpha mr \in \ker b \cap \alpha mR = 0$. So $mr \in \ker\alpha \cap mR$. If $mr_1 \in \ker\alpha \cap mR \Rightarrow \alpha mr_1 = 0 \Rightarrow (b\alpha +s)mr_1 = b\alpha mr_1 + smr_1 = bb\alpha mr_1 = 0 \Rightarrow \alpha mr \in \ker b \cap \alpha mR = 0$. So $b\alpha +s \in S_{(\alpha,m)}$. Hence $S_{(\alpha,m)} = B_{\alpha m}\ \alpha + l_S(M)$

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